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Comparing the practices of primary school mathematics teacher education

Case studies from Japan, Finland and Sweden

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In this study, we have observed three different teacher educators' lessons, concerning area determination of polygons in primary school teacher training courses in Japan, Finland and Sweden. The aim of this paper is to investigate what kind of conditions and constraints influence the construction of the lessons in each country. We focus on the educators' main emphasis of the lessons and analyse the complexities of the mathematical and didactical organisations of each lesson by applying Chevallard's anthropological theory of the didactic (ATD). The analysis shows how the curricula, different traditions of teaching practice in each country impact on the mathematical and didactical construction of the lessons.

Keywords: Teacher education, anthropological theory of the didactic, praxeologies, mathematical organisation, didactical organisation.

Introduction

The lack of essential knowledge that has been clearly shared in the community of teachers in order to practice their profession has been an issue which was discussed in recent years in Sweden; difficult environments for novice teachers to build up their identities as a teacher due to an unclear "teaching culture" in their working places (Palmér, 2013). As an attempt to elucidate the factors that caused the difficulties above in Sweden, we investigate the lesson structures of mathematics education at the primary school teacher education programs, using a comparative study. For comparison, we chose Finland which had significantly high result in mathematical literacy among Scandinavian countries (OECD, 2013); and Japan, where teaching culture in mathematics seems to be more shared, compared to the US and Europe (Winsløw, 2012).

The aim of this study is to investigate the conditions and constraints influence on teacher educators for constructing their lessons concerning area determination in pre-service teacher education. Our question in this paper is two-fold. First, what is the main emphasis in the design of the three lessons? Second, from what kind of conditions, are those emphases originated, and how these conditions influence on the mathematical and didactical organisations of the lessons in each country?

Theoretical framework and methods

Chevallard proposed to study the mathematical knowledge in an institutional context; learning mathematics is extended to any other human activity and gives rise to the anthropological theory of the didactic (ATD). There, mathematics learning is modelled as the construction of praxeologies

(Bosch & Gascón, 2007) within social institutions. A praxeology provides both methods for the solution of a domain of problems (praxis) and a structure (the logos) for the discourse regarding the methods and their relations to broader settings. The form of a praxeology is determined by a mutual interaction of a mathematical organisation (MO) and a didactical organisation (DO). The MO describes mathematical activities of the praxeology, and the DO describes the activities to support the learning or teaching of the MO. The praxeologies into increasing complexity (Garcia, Gascón, Higueras & Bosch, 2006) are classified as: *specific*, *local* and *regional* praxeologies. A specific praxeology consists of a single type of task, where a specific technique is applied, thus the technology usually is implicit. A local praxeology is characterised by the integration of several specific praxeologies that are connected by a common technology. A regional praxeology is a coordination of local praxeologies where a common theory is framed. The form of a praxeology takes depends upon a structuring schema in several levels in a "hierarchy of levels of co-determination" (Bosch & Gascón, 2006): civilisation/society (e.g. political, or cultural orientation in education), school (e.g. curriculum), pedagogy (e.g. general teaching principles), discipline (e.g. mathematics, physics,…), domain (e.g. algebra, geometry,…), sector (e.g. equations, similarity,…), theme (e.g. triangles, root,…) and subject (e.g. one simple question). These levels generate the conditions and restrictions that influence the granularity and form of praxeologies.

Analytical methods

What is unique with teacher education is, *what* educators teach is also *how* educators teach, and *what* the prospective teachers learn is also *how* they are learning (Liljedahl, Durand-Guerrier, Winsløw, Bloch, Huckstep, Rowland et al., 2009, p. 29). In that sense, we analyse how the construction of the mathematical organisation (MO) and the didactical organisation (DO) are interwoven in the lessons. By analysing the constructions of the praxeologies, we can understand in what way the lesson is designed to support prospective teachers to learn mathematical contents and the same time, how to teach these contents in practice. Our hypothesis is that, if the interweaving of the MO and the DO is clearly observable in a teacher educator's lesson, it indicates that the knowledge about the transposition from the content knowledge to the teaching practice has been explicitly presented for prospective teachers.

To collect the data from the lessons, video recordings were made; "Quantity and Measurement" by Mr. Matsui (Japan, with 55 students), "Area of Polygons" by Mrs. Laine (Finland, with 34 students) and "Area and Perimeter" by Mrs. Nilsson (Sweden, with 20 students). The teacher educator in Finland is the third author of this paper. The names of the educators in Japan and Sweden are pseudonyms.

We also classify the complexities of the praxeologies (the MOs and the DOs) of each teacher educator's lesson as specific, local and regional levels to study mathematical and didactical "richness" of the lessons. To investigate the origin of the conditions and constraints which forms the praxeologies in different levels, we analysed each country's curricula concerning the notion of measurement. Also, questionnaires answered by the three teacher educators were analysed using the "hierarchy of levels of co-determination" of ATD. In the questionnaire, the educators in each country were asked for instance, "what do you intend the students to learn in this content (e.g. area of polygons)?", "which kind of difficulties connected with teaching the content and what teaching procedures do you use to engage with this content?" etc. The teacher educators were asked to answer the questions after they conducted their lessons. Studying the educators' answers to a question such as "What are your teaching procedures and particular reasons for using these to engage your teaching (on e.g. area of polygons)?" gives us a picture of what each educator emphasises when they lecture about e.g. area determination.

Results and analysis

Curriculum concerning measurement in each country

In the Japanese guideline for the curriculum for grades one to six (MEXT, 2008), the determination of length, area and volume is described in a separate chapter *Quantity and Measurements*, between the chapters of *Arithmetic* and *Geometry*. The contents for each grade are described in detail with concrete teaching proposals. As guidelines for teaching methods, it is stressed to build on pupils' previously learned knowledge and their various ways of solving problems. The introduction of the chapter consists of four phases; direct comparison, indirect comparison, comparison using arbitrary objects, comparison using standard units. This order is clearly followed by Japanese textbooks (Miyakawa, 2010).

The content regarding quantities, units and measurement are shortly described in the chapter *Geometry* in both the Finnish curriculum (Utbildningsstyrelsen, 2014) and the Swedish curriculum (Skolverket, 2011). These curricula do not give any practical guidelines for teaching the contents. In Sweden, textbooks are not controlled by the ministry and presentations of these contents in the textbooks for grades 1-3 are often placed in sections covering Arithmetic (e.g. Brorsson, 2013), although the Swedish curriculum introduces these concept in Geometry. Unlike the Japanese curriculum, the four phases of the introduction of the concept of measurements are not known in Sweden, some textbooks introduce direct comparison and comparison using standard units at the same time (ibid.). Also, the problem that corresponds to the indirect comparison is not addressed in most textbooks[1]. Comparing these two contexts, we might state that the Japanese curriculum does not give much space for different interpretations of its contents. It provides a suggestion of a uniform teaching approach for textbook authors and the users. We assume the reason that many Swedish textbook authors locate the section of measurements in the domain of arithmetic, is to enable a natural connection between area calculations and the basic arithmetical operations. It indicates that different textbooks provide different teaching approaches in Sweden.

Lesson observation "Quantity and Measurement" in Japan

Mr. Matsui is the lecturer of the course "Arithmetic Education" for prospective teachers in a state university located in the middle part of Japan. He explains the four phases in the process of pupils learning about measurement by referring to the curriculum guidelines and clarifies those different comparison methods for the class. Thereafter, he discusses how the above mentioned four phases are treated in digital textbooks for grades one to five. The second half of the class is spent to experience the structured problem solving approach. This approach emphasises learner's active

[1] We have not completed the investigation of Finnish textbooks yet.

participation in mathematical activities, challenging problems and collective reflections (Stigler & Hiebert, 1999). Mr. Matsui lets the prospective teachers find out several different methods for the determination of the area of parallelograms aiming to teach pupils of grade five. Four students draw pictures and explain their different solutions on the blackboard (see Figure 1). Mr. Matsui points out the different kinds of "shifts" used by students and categorises them in; using the sum of the squares (Figure 1 in the middle), "*parallel translation*" (top left), "*rotation*" (bottom left), "*same area transformation*" (top left and bottom left) and "*double area transformation*" (top right). Then, he explains the formula for the area of parallelogram as height times length since the geometric transformations shows that the width (or height) and length of the parallelograms corresponds to those of rectangles. In the same way, he gives a final problem to find out methods for determining the area of trapezoids, using same didactical approach, and concludes the formula for the area of trapezoid; (a + b) h/2.

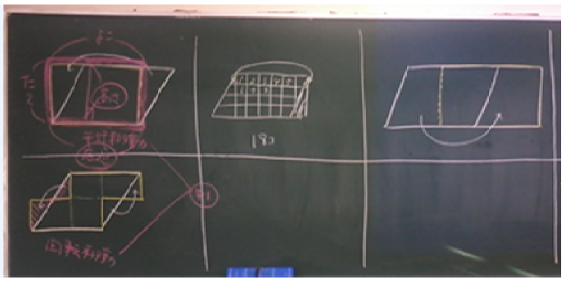


Figure 1: The solutions of the prospective teachers

We now describe in more detail the praxeology of the last part of the lesson demonstrated above.

The mathematical organisation (MO): Types of tasks (**T**): to derive a formula for the area of parallelograms/trapezoids. Techniques (**τ**): transformation of shapes, using formulas. Technology (***θ***): comparison, figures, area-translation, rotation, formulas. Theory (**Θ**): Euclidean geometry, figures and area.

The didactical organisation (DO): (**T**): to experience what the praxeology of the lesson of "Quantities and Measurement" can look like. To anticipate how pupils in grade five would solve area determination of polygons during a lesson, by considering the pupils' previous knowledge. **(τ)**: make the student participate in an exemplary lesson using the structured problem solving approach, and follow it up with whole-class discussions. (***θ***): statement of pupils' previous knowledge about area of rectangles, mathematical textbook and curriculum used as reference. (**Θ**): structured problem solving.

Findings: The first step to realise the didactical task ($\mathbf{T}_d$) mentioned above is, giving the mathematical task ($\mathbf{T}_m$), which leads to a several techniques ($\boldsymbol{\tau}_m$). Those different techniques must be discussed and justified. This is realised by the didactical technique ($\boldsymbol{\tau}_d$), and further, this in turn leads to the use and establishment of a richer technology and theory of the MO. It means, through discussing/comparing their various solving methods, the prospective teachers recognise the area-translation as a fundamental concept to reach algebraic interpretation of area determination. The *logos* part of the DO – didactical technology ($\boldsymbol{\theta}_d$) and the theory ($\boldsymbol{\Theta}_d$) – justify this didactical technique. Thus, the interweaving of the MO and the DO is obvious. Also, the complexity of the praxeology becomes at least local, since several techniques of the different kind of transformation of the shapes are generated by common technology, due to the whole-class discussion.

Lesson observation "Area of Polygons" in Finland

The observed lesson is a workshop using manipulatives in the course "Didactics of Mathematics" for prospective teachers for grades one to six in a state university located in southern Finland. At the last lesson, the lecturer of the course Mrs. Laine has previously explained classification of mathematical figures (e.g. set of squares belong to set of rectangles, and set of rectangles belong to set of parallelograms…), line symmetry and rotational symmetry, perimeter and the area of polygons, property of circle, concept of scale. Today, the prospective teachers move between six different tables to work practically with above mentioned concepts. The students work in groups using a compendium instructing the contents for each table. The compendium is written by Mrs. Laine and she also moves between the tables to give advices to students on to how solve the tasks the compendium suggests. In this paper, we focus on one of the tables "Area of Polygons".

In accordance with the description in the compendium, one prospective teacher in a group plays the "teacher role". As it is prescribed in the compendium, the "teacher" explains how to calculate the area of rectangles by using grid paper with squares of 1cm^2. She says the sum of the squares is equal to the area of the rectangle. In the compendium, it is emphasised that teachers shall promote pupils to use an inductive way of working/learning. It means, letting pupils experience how to calculate the area of different types of rectangles, and have them find out the formula "height times length". The next task is to find out the formula for the area of a parallelogram. The compendium describes the method of parallel translation (however, these didactical terms like *parallel translation* and *same area transformation* were not used in the lesson) and explains that the same formula as for rectangles can be applied. The students explain this method by drawing the figures for their colleagues. In the same way, prospective teachers explain to each other the method of area determination of triangles, by reflecting the instruction of the compendium; "make a parallelogram by drawing two similar triangles and let pupils notice that area of the one of the triangle is the area of the half parallelogram".

The analysis of the mathematical praxeology of the lesson is following:

The MO: Types of tasks (**T**): to derive a formula for the area of rectangles/parallelograms/triangles. Techniques (**τ**): counting the number of the squares, transformation of shapes, using formulas. Technology (**θ**): figures, translation, induction, formulas. Theory (**Θ**): Euclidean geometry, figures and area. The DO: **T**: to experience some teaching method of area determination. **τ**: tasks, compendium, manipulatives, workshop, role-play. **θ**: promoting pupils' inductive thinking **Θ**: lessons with manipulatives/workshop.

Findings: To realise the didactical task ($\mathbf{T}_d$) mentioned above, the mathematical task ($\mathbf{T}_m$) is given to the students. The compendium which is one of the techniques ($\boldsymbol{\tau}_d$) of the DO was referred to find out the mathematical technique ($\boldsymbol{\tau}_m$) and the technologies ($\boldsymbol{\theta}_m$). The didactical technology ($\boldsymbol{\theta}_d$) – promoting pupils' inductive thinking – is mentioned in the compendium even as a mathematical technology. In that way, we may state that the MO and the DO is interwoven in this lesson. However, due to this particular reason – the compendium and role-play give the prospective teachers direct instruction for teaching methods regarding this content – the complexity of the praxeology (the mathematical and the didactical organisation) is specific. The praxeology is generated by always one technique which the educator suggests in the compendium.

Lesson observation "Area and Perimeter" in Sweden

The course Mathematics and Learning for Primary School, Grades 4-6 Teachers II, Geometry, in a state university located in middle of Sweden, treats the knowledge in mathematics and mathematical education in relation to the current Swedish curriculum. The lecturer Mrs. Nilsson gives her students five group-exercises concerning area and perimeter. The sixth exercise consists of determining the area of different geometrical figures by using Geo-board. Mrs. Nilsson demonstrates a method for area-determination of an isosceles triangle by using a rubber band around the triangle. She divides the rectangle into two squares which are in turn divided into two halves. Half of the area of the squares is subtracted from the each side. Now the prospective teachers ponder the method for area-determination of another isosceles triangle in groups.

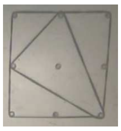
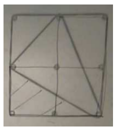
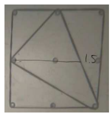
2a 
2b 
2c 

Figure 2a: an isosceles triangle. 2b: with an auxiliary line. 2c: Student B's figure

Mrs. Nilsson then demonstrates student A's solution where the same method is applied as the one she explained. (See Figure 2a & 2b). 4 – 1 – 1 – ½ = 1½ (area units).

Then student B asks if he can apply the formula of the area determination for a triangle. He explains; first, dividing the original triangle into two triangles with the base of 1.5 length units (see Figure 2c), and then adding the area of the two triangles. This gives the area, $(1.5 \cdot 1)/2 + (1.5 \cdot 1)/2 = 0.75 + 0.75 = 1.5$ (area units). Some of the students express that they do not grasp directly how it works. Then Mrs. Nilsson comments "one can understand (this method) if one has more mathematical skills".

Now we sketch the praxeology of the exercise; the determination of the area of an isosceles triangle. The MO: **T**: to determine the area of an isosceles triangle. $\boldsymbol{\tau}$: division of figures and subtraction of area. $\boldsymbol{\theta}$: additivity of area, formula for area determination of rectangles. $\boldsymbol{\Theta}$: Euclidean geometry, figures and area. The DO: **T**: to explore teaching methods for area determination. $\boldsymbol{\tau}$: group discussions about using manipulatives (Geo-board) $\boldsymbol{\theta}$: absent $\boldsymbol{\Theta}$: rules and terminology regarding the use of manipulatives in lessons.

Findings: The didactical task ($\mathbf{T}_d$) and the first step to realise this task (giving the students a mathematical task) is similar to Japan and Finland. However, the didactical technique ($\boldsymbol{\tau}_d$) – using Geo-board – ensures actually several mathematical techniques more than Mrs. Nilsson has planned. This caused a breaking of a didactical contract (Brousseau, 1999) when the student B proposed another technique. Mrs. Nilsson's intention was to train students' algorithmic skills with one technique. She let the student B explain his alternative technique, but did not validate it. Her intention was not to discuss the viability of different mathematical techniques for the grade five class but to establish a certain technique which is possible for all prospective teachers to manage.

Also, the absent of the didactical technology (**θ**) shows that the MO and the DO are not tightly interwoven. The complexity of the praxeology of this lesson remains to be specific, since the task is constructed to be explained by a single technique.

The questionnaires

The Japanese educator states that prospective teachers should learn that the area of geometrical figures, as well as formulas to generalize the calculation of the area, can be determined in various ways, by using pupil's previously learned knowledge. He stresses also that the prospective teachers should be able to use some mathematical terms; the terms describe the various methods for area determination. Further, he comments that he is not concerned about the prospective teachers' mathematical knowledge, since they have passed the entrance exam including comparatively advanced upper secondary school mathematics to be admitted to the teacher education program.

The Finnish educator's intention for the prospective teachers to learn about the area of polygons is that area determination of parallelogram is based on area of rectangles, and area of triangles is based on area of parallelograms. She emphasizes for students to learn the importance of the application of inductive ways of working to find a general result, by examining a number of specific examples. She describes her prospective teachers' fragmental knowledge of the formulas for area determination. During her lesson, she often discusses pupils' misconceptions of area and perimeter to let the prospective teachers realise their own misconceptions of this content.

The Swedish educator mentions the prospective teachers' difficulties and limitations concerning geometrical figures, that some of them have learnt formula for area determination superficially and sometimes incorrectly. Also their perception that "Geometry is a difficult subject" blocks their learning process. Furthermore, the students have not developed mathematical terms that allow them to explain their solutions. To deal with these difficulties, she uses manipulatives; let the prospective teachers work in small groups and becoming confident with just one technique.

Final remarks

The detailed Japanese curriculum which gives a lot of specifications about the teaching approach, the tradition of the structured problem solving and the textbooks adopting the same teaching approaches □ these factors contribute to give practical hints about how to design the lessons with epistemologically well composed praxeologies to a Japanese teacher educator. It becomes explicit for the prospective teachers how to construct mathematics lessons in which alternative techniques are assessed and a technological discourse is taking place. Also, a professional language that describes mathematical techniques such as "same area transformation" is *collectively* used, leading to the knowledge of a teaching practice being *shared* by the community of teacher educators. The conditions forming the scale of the Japanese praxeology originate from the mathematical level, that is the *domain* and the *sector* level. Those conditions are also influenced by even the higher level □ *school* (the curriculum). In Finland, the explicit interweaving of the MO and the DO indicates that the transposition of content knowledge to the teaching practice is explicitly presented for prospective teachers. According to the questionnaire, the Finnish educator's approach aims to stimulate prospective teachers' cognitive learning. Considering the fact that the Finnish curriculum does not give much detailed proposal for the teaching approach to design lessons, we would state that above mentioned condition originates from the *pedagogy* level. It explains why Finnish approach with the tradition of the lessons with workshops does not promote the construction a local or regional praxeology. In the Swedish lesson, the MO and the DO are not interwoven. It seems that presentation of the knowledge transposition is *individually* constructed by teacher educators in Sweden. The result from the questionnaire shows that prospective teachers' fragmented mathematical knowledge and their anxiety for applying mathematics form Mrs. Nilsson's teaching strategies. Thus it is stated that the Swedish condition originate at the *pedagogy* level. Same as in the Finnish case, neither the Swedish curriculum nor the traditions of the lessons with manipulatives promote the construction of complex praxeologies in the lessons for prospective teachers.

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